arXiv:hep-ph/9912425v1 20 Dec 1999

Lifetimes of doubly heavy baryons

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Abstract

We perform a detailed investigation of total lifetimes for the doubly heavy baryons $\Xi_{QQ'}$, $\Omega_{QQ'}$ in the framework of operator product expansion over the inverse heavy quark mass, whereas, to estimate matrix elements of operators obtained in OPE, approximations of nonrelativistic QCD are used.

1 Introduction

At present a number of powerful techniques based on Operator Product Expansion (OPE) and effective field theories have been developed. These tools allow one consistently to include into consideration various nonperturbative contributions, written in terms of a few number of universal quantities. The coefficients (Wilson coefficients) in front of these operators are generally expanded in series over the QCD coupling constant, inverse heavy quark mass and/or relative velocity of heavy quarks inside the hadron. The accuracy, obtained in such calculations, can be systematically improved, and it is limited only by the convergence properties of the mentioned series. The described approach have been already widely used for making the precise predictions in the heavy quark sector of Standard Model (SM), such as decays, distributions and partial width asymmetries involving the CP violation[1] for the heavy hadrons. The sensitivity of Wilson coefficients to virtual corrections caused by some higher-scale interactions makes this approach to be invaluable in searching for a "new" physics at forthcoming experiments.

The approach under discussion has been successfully used in the description of weak decays of the hadrons containing a single heavy quark, as carried out in the framework of Heavy Quark Effective Theory (HQET) [2], in the annihilation and radiative decays of heavy quarkonia $Q\bar{Q}$, where one used the framework of non-relativistic QCD (NRQCD) [3], and in the weak decays

[1] For review see [1].

of long-lived heavy quarkonium with mixed flavours B_c^+ [4] [2]. The experimental data on the weak decays of heavy hadrons can be used for the determination of basic properties of weak interactions at a fundamental level, in particular, for the extraction of CKM matrix elements. The same approach is also valid for the baryons containing two heavy quarks.

In addition to the information extracted from the analysis of hadrons with a single heavy flavor, the baryons with two heavy quarks, $QQ'q$, provide a way to explore the nonspectator effects, where their importance is increased. Here we would like to note, that in the case of systems with two heavy quarks, the hypothesis on the quark-hadron duality is more justified, and, so, the results of OPE-based approach turn out to be more reliable. For these baryons we can apply a method, based on the combined HQET-NRQCD techniques [2, 3, 4], if we use the quark-diquark picture for the bound states. The expansion in the inverse heavy quark mass for the heavy diquark QQ' is a straightforward generalization of these techniques in the mesonic decays of B_c [3, 4], with the difference that, instead of the color-singlet systems, we deal with the color–anti-triplet ones, with the appropriate account for the interaction with the light quark. First estimates of the lifetimes for the doubly heavy baryons $\Xi_{cc}^{\diamond}$ and $\Xi_{bc}^{\diamond}$ were recently performed in [7, 8]. Using the same approach, but different values of parameters[3] a repetition of our results for the case of doubly charmed baryons was done in [10]. The spectroscopic characteristics of baryons with two heavy quarks and the mechanisms of their production in different interactions were discussed in refs. [11, 12, 13, 14] and [15], respectively.

In this paper, we present the calculation of lifetimes for the doubly heavy baryons as well as reconsider the previous estimates with a use of slightly different set of parameters adjusted in the consideration of lifetime data for the observed heavy hadrons and improved spectroscopic inputs. As we made in the description of inclusive decays of the $\Xi_{cc}^{\diamond}$ and $\Xi_{bc}^{\diamond}$-baryons, we follow the papers [4, 16], where all necessary generalizations to the case of hadrons with two heavy quarks and other corrections are discussed. We note, that in the leading order of OPE expansion, the inclusive widths are determined by the mechanism of spectator decays involving free quarks, wherein the corrections in the perturbative QCD are taken into account. The introduction of subleading terms in the expansion over the inverse heavy quark masses[4] allows one to take into account the corrections due to the quark confinement inside the hadron. Here, an essential role is played by both the motion of heavy quark inside the hadron and chromomagnetic interactions of quarks. The important ingredient of such corrections in the baryons with two heavy quarks is the presence of a compact heavy diquark, which implies that the square of heavy quark momentum is enhanced in comparison with the corresponding value for the hadrons with a single heavy quark. The next characteristic feature of baryons with two heavy quarks is the significant numerical impact on the lifetimes by the quark contents of hadrons, since in the third order over the inverse heavy quark mass, $1/m_Q^3$, the four-quark correlations in the total width are enforced in the effective lagrangian due to the two-particle phase space of intermediate states (see the discussion in [16]). In this situation, we have to add the effects of Pauli interference between the products of heavy quark decays and the quarks in the initial state as well as the weak scattering involving the quarks composing the hadron. Due

[2]The first experimental observation of the B_c-meson was recently reported by the CDF Collaboration [5]; see ref.[6] for a theoretical review of B_c-meson physics before the observation.

[3]See comments on the difference in the numerical values of lifetimes of doubly charmed baryons in [9].

[4]It was shown in [17] that the first order $1/m_Q$-correction is absent, and the corrections begin with the $1/m_Q^2$-terms.

to such terms we introduce the corrections depending on spectators and involving the masses of light and strange quarks in the framework of non-relativistic models with the constituent quarks, because they determine the effective physical phase spaces, strongly deviating from the naive estimates in th decays of charmed quarks. We take into account the corrections to the effective weak lagrangian due to the evolution of Wilson coefficients from the scale of the order of heavy quark mass to the energy, characterizing the binding of quarks inside the hadron [16].

The paper is organized as follows. In agreement with the general picture given above, we describe the scheme for the construction of OPE for the total width of baryons containing two heavy quarks with account of corrections to the spectator widths in Section 2. The procedure for the estimation of non-perturbative matrix elements of operators in the doubly heavy baryons is considered in Section 3 in terms of non-relativistic heavy quarks. Section 4 is devoted to the numerical evaluation and discussion of parameter dependence of lifetimes of doubly heavy baryons. We conclude in section 5 by summarizing our results.

2 Description of the method

In this section we describe the approach used for the calculation of total lifetimes for the doubly heavy baryons, originally formulated in [7, 8], together with some new formulae, required for the evaluation of nonspectator effects in the decays of other[5] baryons in the family of doubly heavy baryons, not considered previously.

The optical theorem along with the hypothesis of integral quark-hadron duality, leads us to a relation between the total decay width of heavy quark and the imaginary part of its forward scattering amplitude. This relationship, applied to the $\Xi^{(*)}_{QQ'}$-baryon total decay width $\Gamma_{\Xi^{(*)}_{QQ'}}$, can be written down as:

$$\Gamma_{\Xi^{(*)}_{QQ'}} = \frac{1}{2M_{\Xi^{(*)}_{QQ'}}}\langle\Xi^{(*)}_{QQ'}|\mathcal{T}|\Xi^{(*)}_{QQ'}\rangle, \tag{1}$$

where the $\Xi^{(*)}_{QQ'}$ state in Eq. (1) has the ordinary relativistic normalization, $\langle\Xi^{(*)}_{QQ'}|\Xi^{(*)}_{QQ'}\rangle = 2EV$, and the transition operator $\mathcal{T}$ is determined by the expression

$$\mathcal{T} = \Im m \int d^4x \; \{\hat{T} H_{eff}(x) H_{eff}(0)\}, \tag{2}$$

where H_{eff} is the standard effective hamiltonian, describing the low energy interactions of initial quarks with the decays products, so that

$$H_{eff} = \frac{G_F}{2\sqrt{2}} V_{q_3 q_4} V^*_{q_1 q_2} [C_+(\mu) O_+ + C_-(\mu) O_-] + h.c. \tag{3}$$

where

$$O_\pm = [\bar{q}_{1\alpha}\gamma_\nu(1-\gamma_5)q_{2\beta}][\bar{q}_{3\gamma}\gamma^\nu(1-\gamma_5)q_{4\delta}](\delta_{\alpha\beta}\delta_{\gamma\delta} \pm \delta_{\alpha\delta}\delta_{\gamma\beta}),$$

and

$$C_+ = \left[\frac{\alpha_s(M_W)}{\alpha_s(\mu)}\right]^{\frac{6}{33-2f}}, \quad C_- = \left[\frac{\alpha_s(M_W)}{\alpha_s(\mu)}\right]^{\frac{-12}{33-2f}},$$

[5] Others mean those of not considered in [7, 8].

where f is the number of flavors, $\{\alpha, \beta, \gamma, \delta\}$ run over the color indeces.

Under an assumption, that the energy release in the heavy quark decay is large, we can perform the operator product expansion for the transition operator $\mathcal{T}$ in Eq.(1). In this way we obtain series of local operators with increasing dimensions over the energy scale, wherein the contributions to $\Gamma_{\Xi^{(*)}_{QQ'}}$ are suppressed by the increasing inverse powers of the heavy quark masses. This formalism has already been applied to calculate the total decay rates for the hadrons, containing a single heavy quark [17] (for the most early work, having used similar methods, see also [16, 18]) and hadrons, containing two heavy quarks [7, 8]. As was already pointed in [7], the expansion, applied here, is simultaneously in the powers of both inverse heavy quark masses and the relative velocity of heavy quarks inside the hadron. Thus, this fact shows the difference between the description for the doubly heavy baryons and the consideration of both the heavy-light mesons (the expansion in powers of $\frac{\Lambda_{QCD}}{m_Q}$) and the heavy-heavy mesons [4] (the expansion in powers of relative velocity of heavy quarks inside the hadron, where one can apply the scaling rules of nonrelativistic QCD [19]).

The operator product expansion explored has the form:

$$\mathcal{T} = \sum_{i=1}^{2} C_1(\mu)\bar{Q}^i Q^i + \frac{1}{m_{Q^i}^2} C_2(\mu)\bar{Q}^i g\sigma_{\mu\nu}G^{\mu\nu}Q^i + \frac{1}{m_{Q^i}^3} O(1) \tag{4}$$

The leading contribution in Eq.(4) is determined by the operators $\bar{Q}^i Q^i$, corresponding to the spectator decay of Q^i-quarks. The use of motion equation for the heavy quark fields allows one to eliminate some redundant operators, so that no operators of dimension four contribute. There is a single operator of dimension five, $Q^i_{GQ} = \bar{Q}^i g\sigma_{\mu\nu}G^{\mu\nu}Q^i$. As we will show below, significant contributions come from the operators of dimension six $Q_{2Q^i2q} = \bar{Q}^i\Gamma q\bar{q}\Gamma' Q^i$, representing the effects of Pauli interference and weak scattering for doubly heavy baryons. Furthermore, there are also other operators of dimension six $Q_{61Q^i} = \bar{Q}^i\sigma_{\mu\nu}\gamma_l D^\mu G^{\nu l}Q^i$ and $Q_{62Q^i} = \bar{Q}^i D_\mu G^{\mu\nu}\Gamma_\nu Q^i$, which are suppressed in comparison with Q_{2Q^i2q} [16]. In what follows, we do not calculate the corresponding coefficient functions for the latter two operators, so that the expansion is certainly complete up to the second order of $\frac{1}{m}$, only.

Further, the different contributions to OPE are given by the following:

$$\begin{aligned}
\mathcal{T}_{\Xi_{cc}^{++}} &= 2\mathcal{T}_{35c} + \mathcal{T}^{(1)}_{6,PI}, \\
\mathcal{T}_{\Xi_{cc}^{+}} &= 2\mathcal{T}_{35c} + \mathcal{T}^{(2)}_{6,WS}, \\
\mathcal{T}_{\Omega_{cc}^{+}} &= 2\mathcal{T}_{35c} + \mathcal{T}^{(3)}_{6,PI}, \\
\mathcal{T}_{\Xi_{bc}^{+}} &= \mathcal{T}_{35b} + \mathcal{T}_{35c} + \mathcal{T}^{(4)}_{6,PI} + \mathcal{T}^{(4)}_{6,WS}, \\
\mathcal{T}_{\Xi_{bc}^{0}} &= \mathcal{T}_{35b} + \mathcal{T}_{35c} + \mathcal{T}^{(5)}_{6,PI} + \mathcal{T}^{(5)}_{6,WS}, \\
\mathcal{T}_{\Omega_{bc}^{0}} &= \mathcal{T}_{35b} + \mathcal{T}_{35c} + \mathcal{T}^{(6)}_{6,PI} + \mathcal{T}^{(6)}_{6,WS}, \\
\mathcal{T}_{\Xi_{bb}^{0}} &= 2\mathcal{T}_{35b} + \mathcal{T}^{(7)}_{6,WS}, \\
\mathcal{T}_{\Xi_{bb}^{-}} &= 2\mathcal{T}_{35b} + \mathcal{T}^{(8)}_{6,PI}, \\
\mathcal{T}_{\Omega_{bb}^{-}} &= 2\mathcal{T}_{35b} + \mathcal{T}^{(9)}_{6,PI},
\end{aligned}$$

where the 35-labelled terms account for the operators of dimension three O_{3Q^i} and five O_{GQ^i}, the 6-marked terms correspond to the effects of Pauli interference and weak scattering. The explicit formulae for these contributions have the following form:

$$\mathcal{T}_{35b} = \Gamma_{b,spec}\bar{b}b - \frac{\Gamma_{0b}}{m_b^2}[2P_{c1} + P_{c\tau 1} + K_{0b}(P_{c1} + P_{cc1}) + K_{2b}(P_{c2} + P_{cc2})]O_{Gb}, \tag{5}$$

$$\mathcal{T}_{35c} = \Gamma_{c,spec}\bar{c}c - \frac{\Gamma_{0c}}{m_c^2}[(2 + K_{0c})P_{s1} + K_{2c}P_{s2}]O_{Gc}, \tag{6}$$

where

$$\Gamma_{0b} = \frac{G_F^2 m_b^5}{192\pi^3}|V_{cb}|^2 \qquad , \Gamma_{0c} = \frac{G_F^2 m_c^5}{192\pi^3} \tag{7}$$

with $K_{0Q} = C_-^2 + 2C_+^2$, $K_{2Q} = 2(C_+^2 - C_-^2)$, and $\Gamma_{Q,spec}$ denotes the spectator width (see [17, 20, 21, 22]):

$$P_{c1} = (1-y)^4, \quad P_{c2} = (1-y)^3, \tag{8}$$

$$\begin{aligned} P_{c\tau 1} &= \sqrt{1 - 2(r+y) + (r-y)^2}[1 - 3(r+y) + 3(r^2+y^2) - r^3 - y^3 - 4ry + \\ &\quad 7ry(r+y)] + 12r^2y^2 \ln \frac{(1 - r - y + \sqrt{1 - 2(r+y) + (r-y)^2})^2}{4ry} \end{aligned} \tag{9}$$

$$P_{cc1} = \sqrt{1-4y}(1 - 6y + 2y^2 + 12y^3)24y^4 \ln \frac{1 + \sqrt{1-4y}}{1 - \sqrt{1-4y}} \tag{10}$$

$$P_{cc2} = \sqrt{1-4y}(1 + \frac{y}{2} + 3y^2) - 3y(1 - 2y^2) \ln \frac{1 + \sqrt{1-4y}}{1 - \sqrt{1-4y}} \tag{11}$$

where $y = \frac{m_c^2}{m_b^2}$ and $r = m_\tau^2/m_b^2$. The functions $P_{s1}(P_{s2})$ can be obtained from $P_{c1}(P_{c2})$ by the substitution $y = m_s^2/m_c^2$. In the b-quark decays, we neglect the value m_s^2/m_b^2 and suppose $m_s = 0$.

The calculation of both the Pauli interference effect for the products of heavy quark decays with the quarks in the initial state and the weak scattering of quarks, composing the hadron, depends on the quark contents of baryons and results in:

$$\mathcal{T}_{6,PI}^{(1)} = 2\mathcal{T}_{PI,u\bar{d}}^c \tag{12}$$

$$\mathcal{T}_{6,WS}^{(2)} = 2\mathcal{T}_{WS,cd} \tag{13}$$

$$\mathcal{T}_{6,PI}^{(3)} = 2\mathcal{T}_{PI,u\bar{d}}^{c'} + 2\sum_l \mathcal{T}_{PI,\nu_l\bar{l}}^c \tag{14}$$

$$\mathcal{T}_{6,PI}^{(4)} = \mathcal{T}_{PI,u\bar{d}}^c + \mathcal{T}_{PI,s\bar{c}}^b + \mathcal{T}_{PI,d\bar{u}}^b + \sum_l \mathcal{T}_{PI,l\bar{\nu}_l}^b \tag{15}$$

$$\mathcal{T}_{6,WS}^{(4)} = \mathcal{T}_{WS,bu} + \mathcal{T}_{WS,bc} \tag{16}$$

$$\mathcal{T}_{6,PI}^{(5)} = \mathcal{T}_{PI,s\bar{c}}^b + \mathcal{T}_{PI,d\bar{u}}^b + \mathcal{T}_{PI,d\bar{u}}^{\prime b} + \sum_l \mathcal{T}_{PI,l\bar{\nu}_l}^b \tag{17}$$

$$\mathcal{T}_{6,WS}^{(5)} = \mathcal{T}_{WS,cd} + \mathcal{T}_{WS,bc} \tag{18}$$

$$\begin{aligned}
\mathcal{T}^{(6)}_{6,PI} &= \mathcal{T}^{c'}_{PI,u\bar{d}} + \sum_l \mathcal{T}^{c}_{PI,\nu_l\bar{l}} + \mathcal{T}^{b}_{PI,s\bar{c}} + \mathcal{T}^{b}_{PI,d\bar{u}} + \sum_l \mathcal{T}^{b}_{PI,l\bar{\nu}_l} + \mathcal{T}'^{b}_{PI,s\bar{c}} && (19)\\
\mathcal{T}^{(6)}_{6,WS} &= \mathcal{T}_{WS,bc} + \mathcal{T}_{WS,cs} && (20)\\
\mathcal{T}^{(7)}_{6,WS} &= 2\mathcal{T}_{WS,bu} && (21)\\
\mathcal{T}^{(8)}_{6,PI} &= 2\mathcal{T}'^{b}_{PI,d\bar{u}} && (22)\\
\mathcal{T}^{(9)}_{6,PI} &= 2\mathcal{T}'^{b}_{PI,s\bar{c}} && (23)
\end{aligned}$$

so that

$$\begin{aligned}
\mathcal{T}^{b}_{PI,s\bar{c}} &= -\frac{G_F^2|V_{cb}|^2}{4\pi} m_b^2 (1-\frac{m_c}{m_b})^2 \\
&([(\frac{(1-z_-)^2}{2} - \frac{(1-z_-)^3}{4})(\bar{b}_i\gamma_\alpha(1-\gamma_5)b_i)(\bar{c}_j\gamma^\alpha(1-\gamma_5)c_j) + \\
&(\frac{(1-z_-)^2}{2} - \frac{(1-z_-)^3}{3})(\bar{b}_i\gamma_\alpha\gamma_5 b_i)(\bar{c}_j\gamma^\alpha(1-\gamma_5)c_j)] && (24)\\
&[(C_+ - C_-)^2 + \frac{1}{3}(1-k^{\frac{1}{2}})(5C_+^2 + C_-^2 + 6C_-C_+)] + \\
&[(\frac{(1-z_-)^2}{2} - \frac{(1-z_-)^3}{4})(\bar{b}_i\gamma_\alpha(1-\gamma_5)b_j)(\bar{c}_j\gamma^\alpha(1-\gamma_5)c_i) + \\
&(\frac{(1-z_-)^2}{2} - \frac{(1-z_-)^3}{3})(\bar{b}_i\gamma_\alpha\gamma_5 b_j)(\bar{c}_j\gamma^\alpha(1-\gamma_5)c_i)]k^{\frac{1}{2}}(5C_+^2 + C_-^2 + 6C_-C_+)), \\
\mathcal{T}^{b}_{PI,\tau\bar{\nu}_\tau} &= -\frac{G_F^2|V_{cb}|^2}{\pi} m_b^2 (1-\frac{m_c}{m_b})^2 \\
&[(\frac{(1-z_\tau)^2}{2} - \frac{(1-z_\tau)^3}{4})(\bar{b}_i\gamma_\alpha(1-\gamma_5)b_j)(\bar{c}_j\gamma^\alpha(1-\gamma_5)c_i) + && (25)\\
&(\frac{(1-z_\tau)^2}{2} - \frac{(1-z_\tau)^3}{3})(\bar{b}_i\gamma_\alpha\gamma_5 b_j)(\bar{c}_j\gamma^\alpha(1-\gamma_5)c_i)], \\
\mathcal{T}^{b'}_{PI,d\bar{u}} &= -\frac{G_F^2|V_{cb}|^2}{4\pi} m_b^2 (1-\frac{m_d}{m_b})^2 \\
&([(\frac{(1-z_-)^2}{2} - \frac{(1-z_-)^3}{4})(\bar{b}_i\gamma_\alpha(1-\gamma_5)b_i)(\bar{d}_j\gamma^\alpha(1-\gamma_5)d_j) + \\
&(\frac{(1-z_-)^2}{2} - \frac{(1-z_-)^3}{3})(\bar{b}_i\gamma_\alpha\gamma_5 b_i)(\bar{d}_j\gamma^\alpha(1-\gamma_5)d_j)] && (26)\\
&[(C_+ + C_-)^2 + \frac{1}{3}(1-k^{\frac{1}{2}})(5C_+^2 + C_-^2 - 6C_-C_+)] + \\
&[(\frac{(1-z_-)^2}{2} - \frac{(1-z_-)^3}{4})(\bar{b}_i\gamma_\alpha(1-\gamma_5)b_j)(\bar{d}_j\gamma^\alpha(1-\gamma_5)d_i) + \\
&(\frac{(1-z_-)^2}{2} - \frac{(1-z_-)^3}{3})(\bar{b}_i\gamma_\alpha\gamma_5 b_j)(\bar{d}_j\gamma^\alpha(1-\gamma_5)d_i)]k^{\frac{1}{2}}(5C_+^2 + C_-^2 - 6C_-C_+)), \\
\mathcal{T}^{b'}_{PI,s\bar{c}} &= -\frac{G_F^2|V_{cb}|^2}{16\pi} m_b^2 (1-\frac{m_s}{m_b})^2 \sqrt{(1-4z_-)} \\
&([(1-z_-)(\bar{b}_i\gamma_\alpha(1-\gamma_5)b_i)(\bar{s}_j\gamma^\alpha(1-\gamma_5)s_j) + \\
&\frac{2}{3}(1+2z_-)(\bar{b}_i\gamma_\alpha\gamma_5 b_i)(\bar{s}_j\gamma^\alpha(1-\gamma_5)s_j)] && (27)
\end{aligned}$$

$$
\begin{aligned}
& [(C_+ + C_-)^2 + \frac{1}{3}(1-k^{\frac{1}{2}})(5C_+^2 + C_-^2 - 6C_-C_+)] + \\
& [(1-z_-)(\bar{b}_i\gamma_\alpha(1-\gamma_5)b_j)(\bar{s}_j\gamma^\alpha(1-\gamma_5)s_i) + \\
& \frac{2}{3}(1+2z_-)(\bar{b}_i\gamma_\alpha\gamma_5 b_j)(\bar{s}_j\gamma^\alpha(1-\gamma_5)s_i)]k^{\frac{1}{2}}(5C_+^2 + C_-^2 - 6C_-C_+)),
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{T}^c_{PI,u\bar{d}} = & -\frac{G_F^2}{4\pi}m_c^2(1-\frac{m_u}{m_c})^2 \\
& ([(\frac{(1-z_-)^2}{2} - \frac{(1-z_-)^3}{4})(\bar{c}_i\gamma_\alpha(1-\gamma_5)c_i)(\bar{u}_j\gamma^\alpha(1-\gamma_5)u_j) + \\
& (\frac{(1-z_-)^2}{2} - \frac{(1-z_-)^3}{3})(\bar{c}_i\gamma_\alpha\gamma_5 c_i)(\bar{u}_j\gamma^\alpha(1-\gamma_5)u_j)] \\
& [(C_+ + C_-)^2 + \frac{1}{3}(1-k^{\frac{1}{2}})(5C_+^2 + C_-^2 - 6C_-C_+)] + \\
& [(\frac{(1-z_-)^2}{2} - \frac{(1-z_-)^3}{4})(\bar{c}_i\gamma_\alpha(1-\gamma_5)c_j)(\bar{u}_j\gamma^\alpha(1-\gamma_5)u_i) + \\
& (\frac{(1-z_-)^2}{2} - \frac{(1-z_-)^3}{3})(\bar{c}_i\gamma_\alpha\gamma_5 c_j)(\bar{u}_j\gamma^\alpha(1-\gamma_5)u_i)]k^{\frac{1}{2}}(5C_+^2 + C_-^2 - 6C_-C_+)),
\end{aligned} \tag{28}
$$

$$
\begin{aligned}
\mathcal{T}^{c'}_{PI,u\bar{d}} = & -\frac{G_F^2}{4\pi}m_c^2(1-\frac{m_s}{m_c})^2 \\
& ([\frac{1}{4}(\bar{c}_i\gamma_\alpha(1-\gamma_5)c_i)(\bar{s}_j\gamma^\alpha(1-\gamma_5)s_j) + \frac{1}{6}(\bar{c}_i\gamma_\alpha\gamma_5 c_i)(\bar{s}_j\gamma^\alpha(1-\gamma_5)s_j)] \\
& [(C_+ - C_-)^2 + \frac{1}{3}(1-k^{\frac{1}{2}})(5C_+^2 + C_-^2 + 6C_-C_+)] + \\
& [\frac{1}{4}(\bar{c}_i\gamma_\alpha(1-\gamma_5)c_j)(\bar{s}_j\gamma^\alpha(1-\gamma_5)s_i) + \\
& \frac{1}{6}(\bar{c}_i\gamma_\alpha\gamma_5 c_j)(\bar{s}_j\gamma^\alpha(1-\gamma_5)s_i)]k^{\frac{1}{2}}(5C_+^2 + C_-^2 + 6C_-C_+)),
\end{aligned} \tag{29}
$$

$$
\begin{aligned}
\mathcal{T}^c_{PI,\nu_\tau\bar{\tau}} = & -\frac{G_F^2}{\pi}m_c^2(1-\frac{m_s}{m_c})^2 \\
& [(\frac{(1-z_\tau)^2}{2} - \frac{(1-z_\tau)^3}{4})(\bar{c}_i\gamma_\alpha(1-\gamma_5)c_j)(\bar{s}_j\gamma^\alpha(1-\gamma_5)s_i) + \\
& (\frac{(1-z_\tau)^2}{2} - \frac{(1-z_\tau)^3}{3})(\bar{c}_i\gamma_\alpha\gamma_5 c_j)(\bar{s}_j\gamma^\alpha(1-\gamma_5)s_i)],
\end{aligned} \tag{30}
$$

$$
\begin{aligned}
\mathcal{T}_{WS,bc} = & \frac{G_F^2|V_{cb}|^2}{4\pi}m_b^2(1+\frac{m_c}{m_b})^2(1-z_+)^2[(C_+^2 + C_-^2 + \frac{1}{3}(1-k^{\frac{1}{2}})(C_+^2 - C_-^2)) \\
& (\bar{b}_i\gamma_\alpha(1-\gamma_5)b_i)(\bar{c}_j\gamma^\alpha(1-\gamma_5)c_j) + \\
& k^{\frac{1}{2}}(C_+^2 - C_-^2)(\bar{b}_i\gamma_\alpha(1-\gamma_5)b_j)(\bar{c}_j\gamma^\alpha(1-\gamma_5)c_i)],
\end{aligned} \tag{31}
$$

$$
\begin{aligned}
\mathcal{T}_{WS,bu} = & \frac{G_F^2|V_{cb}|^2}{4\pi}m_b^2(1+\frac{m_u}{m_b})^2(1-z_+)^2[(C_+^2 + C_-^2 + \frac{1}{3}(1-k^{\frac{1}{2}})(C_+^2 - C_-^2)) \\
& (\bar{b}_i\gamma_\alpha(1-\gamma_5)b_i)(\bar{u}_j\gamma^\alpha(1-\gamma_5)u_j) + \\
& k^{\frac{1}{2}}(C_+^2 - C_-^2)(\bar{b}_i\gamma_\alpha(1-\gamma_5)b_j)(\bar{u}_j\gamma^\alpha(1-\gamma_5)u_i)],
\end{aligned} \tag{32}
$$

$$
\mathcal{T}_{WS,cd} = \frac{G_F^2}{4\pi}m_c^2(1+\frac{m_d}{m_c})^2(1-z_+)^2[(C_+^2 + C_-^2 + \frac{1}{3}(1-k^{\frac{1}{2}})(C_+^2 - C_-^2))
$$

$$(\bar{c}_i\gamma_\alpha(1-\gamma_5)c_i)(\bar{d}_j\gamma^\alpha(1-\gamma_5)d_j) + \tag{33}$$
$$k^{\frac{1}{2}}(C_+^2 - C_-^2)(\bar{c}_i\gamma_\alpha(1-\gamma_5)c_j)(\bar{d}_j\gamma^\alpha(1-\gamma_5)d_i)],$$

$$\mathcal{T}^b_{PI,d\bar{u}} = \mathcal{T}^b_{PI,s\bar{c}}\ (z_- \to 0) \tag{34}$$
$$\mathcal{T}^b_{PI,e\bar{\nu}_e} = \mathcal{T}^b_{PI,\mu\bar{\nu}_\mu} = \mathcal{T}^b_{PI,\tau\bar{\nu}_\tau}\ (z_\tau \to 0), \tag{35}$$
$$\mathcal{T}^c_{PI,\nu_e\bar{e}} = \mathcal{T}^c_{PI,\nu_\mu\bar{\mu}} = \mathcal{T}^c_{PI,\nu_\tau\bar{\tau}}\ (z_\tau \to 0), \tag{36}$$

where the following notation has been used:

$$\text{in Eq. (24)} \quad z_- = \frac{m_c^2}{(m_b-m_c)^2}, \quad k = \frac{\alpha_s(\mu)}{\alpha_s(m_b-m_c)},$$
$$\text{in Eq. (25)} \quad z_\tau = \frac{m_\tau^2}{(m_b-m_c)^2},$$
$$\text{in Eq. (26)} \quad z_- = \frac{m_c^2}{(m_b-m_d)^2}, \quad k = \frac{\alpha_s(\mu)}{\alpha_s(m_b-m_d)},$$
$$\text{in Eq. (27)} \quad z_- = \frac{m_c^2}{(m_b-m_s)^2}, \quad k = \frac{\alpha_s(\mu)}{\alpha_s(m_b-m_s)},$$
$$\text{in Eq. (28)} \quad z_- = \frac{m_s^2}{(m_c-m_u)^2}, \quad k = \frac{\alpha_s(\mu)}{\alpha_s(m_c-m_u)},$$
$$\text{in Eq. (29)} \quad k = \frac{\alpha_s(\mu)}{\alpha_s(m_c-m_s)},$$
$$\text{in Eq. (30)} \quad z_\tau = \frac{m_\tau^2}{(m_c-m_s)^2},$$
$$\text{in Eq. (31)} \quad z_+ = \frac{m_c^2}{(m_b+m_c)^2}, \quad k = \frac{\alpha_s(\mu)}{\alpha_s(m_b+m_c)},$$
$$\text{in Eq. (32)} \quad z_+ = \frac{m_c^2}{(m_b+m_u)^2}, \quad k = \frac{\alpha_s(\mu)}{\alpha_s(m_b+m_u)},$$
$$\text{in Eq. (33)} \quad z_+ = \frac{m_s^2}{(m_c+m_d)^2}, \quad k = \frac{\alpha_s(\mu)}{\alpha_s(m_c+m_d)}.$$

In the evolution of coefficients C_+ and C_-, we have taken into account the threshold effects, connected to the heavy quark masses.

In expressions (5) and (6), the scale μ has been taken approximately equal to m_c. In the Pauli interference term, we suggest that the scale can be determined on the basis of the agreement of the experimentally known difference between the lifetimes of Λ_c, Ξ_c^+ and Ξ_c^0 with the theoretical predictions in the framework described above[6]. In any case, the choice of the normalization scale leads to uncertainties in the final results. Theoretical accuracy can be improved by the calculation of next-order corrections in the powers of QCD coupling constant.

The coefficients of leading terms, represented by operators $\bar{b}b$ and $\bar{c}c$, are equivalent to the widths fot the decays of free quarks and are known in the next-to-leading logarithmic approximation of QCD [23, 24, 25, 26, 27], including the mass corrections in the final state

[6] A more extended description is presented in [7].

with the charmed quark and τ-lepton [27] in the decays of b-quark and with the strange quark mass for the decays of c-quark. In the numerical estimates, we include these corrections and mass effects, but we neglect the decay modes suppressed by the Cabibbo angle, and also the strange quark mass effects in b decays.

The expressions for the contribution of operator $\sum_{i=1}^{2} O_{GQ^i}$ are known in the leading logarithmic approximation. The expressions for the contributions of operators with the dimension 6 have been calculated by us with account of masses in the final states together with the logarithmic renormalization of the effective lagrangian for the non-relativistic heavy quarks at energies less than the heavy quark masses.

Thus, the calculation of lifetimes for the baryons $\Xi^{\diamond}_{QQ'}$ is reduced to the problem of evaluating the matrix elements of operators, which is the subject of next section.

3 Matrix elements in NRQCD approximation.

By using the equations of motion, the matrix element of operator $\bar{Q}^j Q^j$ can be expanded in series over the powers of $1/m_{Q^j}$:

$$\langle \Xi^{\diamond}_{QQ'}|\bar{Q}^j Q^j|\Xi^{\diamond}_{QQ'}\rangle_{norm} = 1 - \frac{\langle \Xi^{\diamond}_{QQ'}|\bar{Q}^j[(i\boldsymbol{D})^2 - (\frac{i}{2}\sigma G)]Q^j|\Xi^{\diamond}_{QQ'}\rangle_{norm}}{2m^2_{Q^j}} + O(\frac{1}{m^3_{Q^j}}). \qquad (37)$$

Thus, we have to estimate the matrix elements of operators from the following list only:

$$\begin{array}{l} \bar{Q}^j(i\boldsymbol{D})^2Q^j, \quad (\frac{i}{2})\bar{Q}^j\sigma G Q^j, \quad \bar{Q}^j\gamma_\alpha(1-\gamma_5)Q^j\bar{q}\gamma^\alpha(1-\gamma_5)q, \\ \bar{Q}^j\gamma_\alpha\gamma_5 Q^j\bar{q}\gamma^\alpha(1-\gamma_5)q, \quad \bar{Q}^j\gamma_\alpha\gamma_5 Q^j\bar{Q}^k\gamma^\alpha(1-\gamma_5)Q^k, \\ \bar{Q}^j\gamma_\alpha(1-\gamma_5)Q^j\bar{Q}^k\gamma^\alpha(1-\gamma_5)Q^k. \end{array} \qquad (38)$$

The meaning of each term in the above list, was already discussed by us in the previous papers on the decays of doubly heavy baryons [7, 8], so we omit it here.

Further, employing the NRQCD expansion of operators $\bar{Q}Q$ and $\bar{Q}g\sigma_{\mu\nu}G^{\mu\nu}Q$, we have

$$\begin{aligned} \bar{Q}Q &= \Psi^\dagger_Q\Psi_Q - \frac{1}{2m^2_Q}\Psi^\dagger_Q(i\boldsymbol{D})^2\Psi_Q + \frac{3}{8m^4_Q}\Psi^\dagger_Q(i\boldsymbol{D})^4\Psi_Q - \\ &\quad \frac{1}{2m^2_Q}\Psi^\dagger_Q g\boldsymbol{\sigma}\boldsymbol{B}\Psi_Q - \frac{1}{4m^3_Q}\Psi^\dagger_Q(\boldsymbol{D}g\boldsymbol{E})\Psi_Q + ... \qquad (39) \\ \bar{Q}g\sigma_{\mu\nu}G^{\mu\nu}Q &= -2\Psi^\dagger_Q g\boldsymbol{\sigma}\boldsymbol{B}\Psi_Q - \frac{1}{m_Q}\Psi^\dagger_Q(\boldsymbol{D}g\boldsymbol{E})\Psi_Q + ... \qquad (40) \end{aligned}$$

Here the factorization at scale μ ($m_Q > \mu > m_Q v_Q$) is supposed. We have omitted the term of $\Psi^\dagger_Q\boldsymbol{\sigma}(g\boldsymbol{E}\times\boldsymbol{D})\Psi_Q$, corresponding to the spin-orbital interactions, which are not essential for the basic state of baryons under consideration. The field Ψ_Q has the standard non-relativistic normalization.

Now we would like to make some comments on the difference between the descriptions of interactions of the heavy quark with the light and heavy heavy ones. As well known, in the doubly heavy subsystem there is an additional parameter which is the relative velocity of

quarks. It introduces the energy scale equal to $m_Q v$. Therefore, the Darwin term ($\boldsymbol{DE}$) in the heavy subsystem stands in the same order of inverse heavy quark mass in comparison with the chromomagnetic term ($\boldsymbol{\sigma B}$) (they have the same power in the velocity v). This statement becomes evident if we apply the scaling rules of NRQCD [19]:

$$\Psi_Q \sim (m_Q v_Q)^{\frac{3}{2}}, \quad |\boldsymbol{D}| \sim m_Q v_Q, \quad gE \sim m_Q^2 v_Q^3, \quad gB \sim m_Q^2 v_Q^4, \quad g \sim v_Q^{\frac{1}{2}}.$$

For the interaction of heavy quark with the light one, there is no such small velocity parameter, so that the Darwin term is suppressed by the additional factor of $k/m_Q \sim \Lambda_{QCD}/m_Q$.

Further, the phenomenological experience with the potential quark models shows, that the kinetic energy of quarks practically does not depend on the quark contents of system, and it is determined by the color structure of state. So, we suppose that the kinetic energy is equal to $T = m_d v_d^2/2 + m_l v_l^2/2$ for the quark-diquark system, and it is $T/2 = m_b v_b^2/2 + m_c v_c^2/2$ in the diquark (the color factor of 1/2). Then

$$\frac{\langle \Xi^\diamond_{QQ'} | \Psi_Q^\dagger (i\boldsymbol{D})^2 \Psi_Q | \Xi^\diamond_{QQ'} \rangle}{2 M_{\Xi^\diamond_{QQ'}} m_Q^2} \simeq v_Q^2 \simeq \frac{2 m_q T}{(m_q + m_{Q'} + m_Q)(m_{Q'} + m_Q)} + \frac{m_{Q'} T}{m_Q (m_Q + m_{Q'})}, \tag{41}$$

$$\frac{\langle \Xi^\diamond_{QQ'} | \Psi_{Q'}^\dagger (i\boldsymbol{D})^2 \Psi_{Q'} | \Xi^\diamond_{QQ'} \rangle}{2 M_{\Xi^\diamond_{QQ'}} m_{Q'}^2} \simeq v_{Q'}^2 \simeq \frac{2 m_q T}{(m_q + m_{Q'} + m_Q)(m_{Q'} + m_Q)} + \frac{m_Q T}{m_{Q'} (m_Q + m_{Q'})}, \tag{42}$$

where the diquark terms dominate certainly. Applying the quark-diquark approximation and relating the matrix element of chromomagnetic interaction of diquark with the light quark to the mass difference between the exited and ground states $M_{\Xi^{\diamond *}_{QQ'}} - M_{\Xi^\diamond_{QQ'}}$, we have

$$\begin{aligned}
\frac{\langle \Xi^\diamond_{cc} | \bar{c} c | \Xi^\diamond_{cc} \rangle}{2 M_{\Xi^\diamond_{cc}}} &= 1 - \frac{1}{2} v_c^2 - \frac{1}{3} \frac{M_{\Xi^{\diamond *}_{cc}} - M_{\Xi^\diamond_{cc}}}{m_c} - \frac{5 g^2}{18 m_c^3} |\Psi(0)|^2 + ... \\
&\approx 1 - 0.073 - 0.025 - 0.009 + \ldots
\end{aligned} \tag{43}$$

$$\begin{aligned}
\frac{\langle \Omega^\diamond_{cc} | \bar{c} c | \Omega^\diamond_{cc} \rangle}{2 M_{\Omega^\diamond_{cc}}} &= 1 - \frac{1}{2} v_c^2 - \frac{1}{3} \frac{M_{\Omega^{\diamond *}_{cc}} - M_{\Omega^\diamond_{cc}}}{m_c} - \frac{5 g^2}{18 m_c^3} |\Psi(0)|^2 + ... \\
&\approx 1 - 0.078 - 0.025 - 0.009 + \ldots
\end{aligned} \tag{44}$$

$$\begin{aligned}
\frac{\langle \Xi^\diamond_{bc} | \bar{c} c | \Xi^\diamond_{bc} \rangle}{2 M_{\Xi^\diamond_{bc}}} &= 1 - \frac{1}{2} v_c^2 + \frac{g^2}{3 m_b m_c^2} |\Psi^d(0)|^2 - \frac{1}{6 m_c^3} g^2 |\Psi^d(0)|^2 + \ldots \\
&\approx 1 - 0.098 + 0.006 - 0.010 \ldots
\end{aligned} \tag{45}$$

$$\begin{aligned}
\frac{\langle \Omega^\diamond_{bc} | \bar{c} c | \Omega^\diamond_{bc} \rangle}{2 M_{\Omega^\diamond_{bc}}} &= 1 - \frac{1}{2} v_c^2 + \frac{g^2}{3 m_b m_c^2} |\Psi^d(0)|^2 - \frac{1}{6 m_c^3} g^2 |\Psi^d(0)|^2 + \ldots \\
&\approx 1 - 0.099 + 0.006 - 0.010 \ldots
\end{aligned} \tag{46}$$

$$\begin{aligned}
\frac{\langle \Xi^\diamond_{bb} | \bar{b} b | \Xi^\diamond_{bb} \rangle}{2 M_{\Xi^\diamond_{bb}}} &= 1 - \frac{1}{2} v_b^2 - \frac{1}{3} \frac{M_{\Xi^{\diamond *}_{bb}} - M_{\Xi^\diamond_{bb}}}{m_b} - \frac{5 g^2}{18 m_b^3} |\Psi(0)|^2 + ... \\
&\approx 1 - 0.021 - 0.003 - 0.002 + \ldots
\end{aligned} \tag{47}$$

$$\begin{aligned}
\frac{\langle \Omega^\diamond_{bb} | \bar{b} b | \Omega^\diamond_{bb} \rangle}{2 M_{\Omega^\diamond_{bb}}} &= 1 - \frac{1}{2} v_b^2 - \frac{1}{3} \frac{M_{\Omega^{\diamond *}_{bb}} - M_{\Omega^\diamond_{bb}}}{m_b} - \frac{5 g^2}{18 m_b^3} |\Psi(0)|^2 + ... \\
&\approx 1 - 0.021 - 0.003 - 0.002 + \ldots
\end{aligned} \tag{48}$$

The numerical values of parameters used in the calculations above are given in the next section. Our presentation here is less detailed than in previous papers [7, 8]. However, we hope, that the interested reader can find there all needed details.

Analogous expressions may be obtained for the matrix elements of operator $Qg\sigma_{\mu\nu}G^{\mu\nu}Q$

$$\frac{\langle\Xi_{cc}^\diamond|\bar{c}g\sigma_{\mu\nu}G^{\mu\nu}c|\Xi_{cc}^\diamond\rangle}{2M_{\Xi_{cc}^\diamond}m_c^2} = -\frac{4}{3}\frac{(M_{\Xi_{cc}^{\diamond *}}-M_{\Xi_{cc}^\diamond})}{m_c}-\frac{7g^2}{9m_c^3}|\Psi^d(0)|^2 \approx -0.124, \quad (49)$$

$$\frac{\langle\Xi_{bc}^\diamond|\bar{c}g\sigma_{\mu\nu}G^{\mu\nu}c|\Xi_{bc}^\diamond\rangle}{2M_{\Xi_{bc}^\diamond}m_c^2} = \frac{4g^2}{3m_b m_c^2}|\Psi^d(0)|^2-\frac{g^2}{3m_c^3}|\Psi^d(0)|^2 \approx 0.005, \quad (50)$$

$$\frac{\langle\Xi_{bb}^\diamond|\bar{b}g\sigma_{\mu\nu}G^{\mu\nu}b|\Xi_{bb}^{(\diamond)}\rangle}{2M_{\Xi_{bb}^\diamond}m_b^2} = -\frac{4}{3}\frac{(M_{\Xi_{bb}^{\diamond *}}-M_{\Xi_{bb}^\diamond})}{m_b}-\frac{7g^2}{9m_b^3}|\Psi^d(0)|^2 \approx -0.189, \quad (51)$$

$$\langle\Omega_{QQ'}|\bar{c}g\sigma_{\mu\nu}G^{\mu\nu}c|\Omega_{QQ'}\rangle = \langle\Xi_{QQ'}^\diamond|\bar{c}g\sigma_{\mu\nu}G^{\mu\nu}c|\Xi_{QQ'}^\diamond\rangle \quad (52)$$

The permutations of quark masses lead to the required expressions for the operators of $\bar{b}b$ and $\bar{b}g\sigma_{\mu\nu}G^{\mu\nu}b$.

For the four quark operators, determining the Pauli interference and the weak scattering, we use the estimates in the framework of non-relativistic potential model [7, 8]:

$$\langle\Xi_{cc}^\diamond|(\bar{c}\gamma_\mu(1-\gamma_5)c)(\bar{q}\gamma^\mu(1-\gamma_5)q)|\Xi_{cc}^\diamond\rangle = 12(m_c+m_q)\cdot|\Psi^{dl}(0)|^2, \quad (53)$$

$$\langle\Xi_{cc}^\diamond|(\bar{c}\gamma_\mu\gamma_5 c)(\bar{q}\gamma^\mu(1-\gamma_5)q)|\Xi_{cc}^\diamond\rangle = 8(m_c+m_q)\cdot|\Psi^{dl}(0)|^2, \quad (54)$$

$$\langle\Omega_{cc}|(\bar{c}\gamma_\mu(1-\gamma_5)c)(\bar{s}\gamma^\mu(1-\gamma_5)s)|\Omega_{cc}\rangle = 12(m_c+m_s)\cdot|\Psi^{dl}(0)|^2, \quad (55)$$

$$\langle\Omega_{cc}|(\bar{c}\gamma_\mu\gamma_5 c)(\bar{s}\gamma^\mu(1-\gamma_5)s)|\Omega_{cc}\rangle = 8(m_c+m_s)\cdot|\Psi^{dl}(0)|^2, \quad (56)$$

$$\langle\Xi_{bc}^\diamond|(\bar{b}\gamma_\mu(1-\gamma_5)b)(\bar{c}\gamma^\mu(1-\gamma_5)c)|\Xi_{bc}^\diamond\rangle = 8(m_b+m_c)\cdot|\Psi^{d}(0)|^2, \quad (57)$$

$$\langle\Xi_{bc}^\diamond|(\bar{b}\gamma_\mu\gamma_5 b)(\bar{c}\gamma^\mu(1-\gamma_5)c)|\Xi_{bc}^\diamond\rangle = 6(m_b+m_c)\cdot|\Psi^{d}(0)|^2, \quad (58)$$

$$\langle\Xi_{bc}^\diamond|(\bar{b}\gamma_\mu(1-\gamma_5)b)(\bar{q}\gamma^\mu(1-\gamma_5)q)|\Xi_{bc}^\diamond\rangle = 2(m_b+m_l)\cdot|\Psi^{dl}(0)|^2, \quad (59)$$

$$\langle\Xi_{bc}^\diamond|(\bar{b}\gamma_\mu\gamma_5 b)(\bar{q}\gamma^\mu(1-\gamma_5)q)|\Xi_{bc}^\diamond\rangle = 0, \quad (60)$$

$$\langle\Xi_{bc}^\diamond|(\bar{c}\gamma_\mu(1-\gamma_5)c)(\bar{q}\gamma^\mu(1-\gamma_5)q)|\Xi_{bc}^\diamond\rangle = 2(m_c+m_l)\cdot|\Psi^{dl}(0)|^2, \quad (61)$$

$$\langle\Xi_{bc}^\diamond|(\bar{c}\gamma_\mu\gamma_5 c)(\bar{q}\gamma^\mu(1-\gamma_5)q)|\Xi_{bc}^\diamond\rangle = 0, \quad (62)$$

$$\langle\Omega_{bc}|(\bar{b}\gamma_\mu(1-\gamma_5)b)(\bar{c}\gamma^\mu(1-\gamma_5)c)|\Omega_{bc}\rangle = 8(m_b+m_c)\cdot|\Psi^{d}(0)|^2, \quad (63)$$

$$\langle\Omega_{bc}|(\bar{b}\gamma_\mu\gamma_5 b)(\bar{c}\gamma^\mu(1-\gamma_5)c)|\Omega_{bc}\rangle = 6(m_b+m_c)\cdot|\Psi^{d}(0)|^2, \quad (64)$$

$$\langle\Omega_{bc}|(\bar{b}\gamma_\mu(1-\gamma_5)b)(\bar{s}\gamma^\mu(1-\gamma_5)s)|\Omega_{bc}\rangle = 2(m_b+m_s)\cdot|\Psi^{dl}(0)|^2, \quad (65)$$

$$\langle\Omega_{bc}|(\bar{b}\gamma_\mu\gamma_5 b)(\bar{s}\gamma^\mu(1-\gamma_5)s)|\Omega_{bc}\rangle = 0, \quad (66)$$

$$\langle\Omega_{bc}|(\bar{c}\gamma_\mu(1-\gamma_5)c)(\bar{s}\gamma^\mu(1-\gamma_5)s)|\Omega_{bc}\rangle = 2(m_c+m_s)\cdot|\Psi^{dl}(0)|^2, \quad (67)$$

$$\langle\Omega_{bc}|(\bar{c}\gamma_\mu\gamma_5 c)(\bar{s}\gamma^\mu(1-\gamma_5)s)|\Omega_{bc}\rangle = 0, \quad (68)$$

$$\langle\Xi_{bb}^\diamond|(\bar{b}\gamma_\mu(1-\gamma_5)b)(\bar{q}\gamma^\mu(1-\gamma_5)q)|\Xi_{bb}^\diamond\rangle = 12(m_b+m_q)\cdot|\Psi^{dl}(0)|^2, \quad (69)$$

$$\langle\Xi_{bb}^\diamond|(\bar{b}\gamma_\mu\gamma_5 b)(\bar{q}\gamma^\mu(1-\gamma_5)q)|\Xi_{bb}^\diamond\rangle = 8(m_b+m_q)\cdot|\Psi^{dl}(0)|^2. \quad (70)$$

The color structure of wave functions leads to the relations

$$\langle\Xi_{QQ'}^\diamond|(\bar{Q}_iT_\mu Q_k)(\bar{q}_k\gamma^\mu(1-\gamma_5)q_i)|\Xi_{QQ'}^\diamond\rangle = -\langle\Xi_{QQ'}^\diamond|(\bar{Q}T_\mu Q)(\bar{q}\gamma^\mu(1-\gamma_5)q)|\Xi_{QQ'}^\diamond\rangle,$$

where T_μ is an arbitrary spinor matrix.

4 Numerical estimates

Performing the numerical calculations of lifetimes for the doubly heavy baryons, we have used the following set of parameters:

$$
\begin{aligned}
m_s = 0.2 \text{ GeV} \quad m_l = 0. \, GeV \quad m_s^* = 0.45 \text{ GeV} \quad m_l^* = 0.3 \text{ GeV} \\
|V_{cs}| = 0.9745 \quad |V_{bc}| = 0.04 \quad T = 0.4 \text{ GeV} \qquad (71) \\
m_c = 1.55 \text{ GeV} \quad m_b = 5.05 \text{ GeV} \qquad (72)
\end{aligned}
$$

The numerical values of diquark wavefunctions at the origin for baryons under consideration are collected in Table 1. The masses of doubly heavy baryons may be found in Table 2.

	Ξ_{cc}^{++}	Ξ_{cc}^{+}	Ω_{cc}^{+}	Ξ_{bc}^{+}	Ξ_{bc}^{0}	Ω_{bc}^{0}	Ξ_{bb}^{0}	Ξ_{bb}^{-}	Ω_{bb}^{-}
$\Psi^d(0)$, GeV$^{\frac{3}{2}}$	0.150	0.150	0.150	0.205	0.205	0.205	0.380	0.380	0.380

Table 1: The values of diquark wavefunctions for the doubly heavy baryons at the origin.

	Ξ_{cc}^{++}	Ξ_{cc}^{+}	Ω_{cc}^{+}	Ξ_{bc}^{+}	Ξ_{bc}^{0}	Ω_{bc}^{0}	Ξ_{bb}^{0}	Ξ_{bb}^{-}	Ω_{bb}^{-}
M, GeV	3.478	3.478	3.578	6.82	6.82	6.92	10.093	10.093	10.193
M^*, GeV	3.61	3.61	3.71	-	-	-	10.193	10.193	10.293

Table 2: The masses of doubly heavy baryons M, and M^* stands for the mass of the baryon with lowest excited state of light quark-diquark system.

The wavefunctions as well as masses for the considered baryons are taken from [11, 12], where their estimates in the non-relativistic model with the Buchmüller-Tye potential were done. The b-quark mass is obtained from the requirement, that for any given value of c-quark mass the theoretically computed B_d-meson lifetime equals to experimentally measured value. This matching condition leads to the following approximate relation

$$m_b = m_c + 3.5 \text{ GeV}. \qquad (73)$$

The c-quark mass is determined from the analogous matching procedure for the B_c-meson lifetime [9]. The m_q^*-values in Eq. (71) represent the constituent masses for the corresponding light quarks, used by us in estimations of hadronic matrix elements.[7] For the value of light quark-diquark function at the origin we assume

$$|\Psi^{dl}(0)|^2 = \frac{2}{3}\frac{f_D^2 M_D k^{-\frac{4}{9}}}{12}, \qquad (74)$$

where $f_D = 170$ MeV. This expression obtained by performing the steps similar to [28, 29] for the derivation of hyperfine splitting in the light quark-diquark system. The factor $k^{-\frac{4}{9}}$

[7] See [7] for details.

	Ξ_{cc}^{++}	Ξ_{cc}^{+}	Ω_{cc}^{+}
$\sum c \to s$, ps^{-1}	3.104	3.104	3.104
PI, ps^{-1}	-0.874	-	0.621
WS, ps^{-1}	-	1.776	-
τ, ps	0.45	0.20	0.27

Table 3: The lifetimes of doubly charmed baryons together with the relative spectator and nonspectator contributions to the total widths.

accounts for the low energy logarithmic renormalization of f_D constant. We have used this equation for all doubly heavy baryons, considered in this paper. Even though we use this relation to compute central values of lifetimes, the precise values of wavefunction parameters are under question, so in the presented results we have allowed for variations.

The renormalization scale μ is chosen in the following way: $\mu_1 = m_b$ and $\mu_2 = m_c$ in the estimates of Wilson coefficients $C_+(\mu)$ and $C_-(\mu)$ for the effective four-fermion weak lagrangian at low energies with the b and c-quarks, correspondingly. For nonspectator effects, which are the Pauli interference and weak scattering of valence quarks, the renormalization scale μ is obtained from the fit of theoretical predictions for the lifetime differences of baryons Λ_c, Ξ_c^+ and Ξ_c^0 over the experimental data.

In Table 3 we present the results of calculations for the doubly charmed baryons. Together with the total lifetimes of these baryons we have shown the relative spectator and nonspectator contributions. From this Table we see the importance of nonspectator effects, producing huge differences in the values of lifetimes. The analogous results for other doubly heavy baryons can be found in Tables 4 and 5.

A small comment concerns with the corrections to the spectator decays of heavy quarks, caused by the motion of heavy quarks inside the hadron and interactions with the light degrees of freedom. The corrections due to the quark-gluon operators of dimension 5 are numerically small [16]. The most important terms come from the kinetic energy of heavy quarks.

In Figs. 1-9 we have shown the dependence of baryons lifetimes from the values of light quark-diquark wavefunctions at the origin. We see quite a different behaviour withthe increase of $|\Psi^{dl}(0)|$-parameter.

	Ξ_{bc}^{+}	Ξ_{bc}^{0}	Ω_{bc}^{0}
$\sum b \to c$, ps^{-1}	0.632	0.632	0.631
$\sum c \to s$, ps^{-1}	1.511	1.511	1.509
PI, ps^{-1}	0.807	0.855	0.979
WS, ps^{-1}	0.653	0.795	1.713
τ, ps	0.28	0.26	0.21

Table 4: The lifetimes of (bcq)-baryons together with the relative spectator and nonspectator contributions to the total widths.

	Ξ_{bb}^{0}	Ξ_{bb}^{-}	Ω_{bb}^{-}
$\sum b \to c$, ps^{-1}	1.254	1.254	1.254
PI, ps^{-1}	-	-0.0130	-0.0100
WS, ps^{-1}	0.0189	-	-
τ, ps	0.79	0.80	0.80

Table 5: The lifetimes of (bbq)-baryons together with the relative spectator and nonspectator contributions to the total widths.

Here, we would like to note, that in this paper we do not give a detail discussion of nonspectator effects on the total lifetimes and semileptonic branching ratios of doubly heavy baryons and promise to fill this gap in one of our subsequent papers [30].

Finally, concerning the uncertainties of the presented estimates, we note that they are mainly determined by the following:

1) The c-quark mass is poorly known, but constrained by the fits to the experimental data, discussed above, can lead to the uncertainty $\frac{\delta\Gamma}{\Gamma} \approx 15\%$ in the case of doubly charmed baryons and $\frac{\delta\Gamma}{\Gamma} \approx 10\%$ for the case of bcq - baryons.

2) The uncertainties in the values of diquark and light quark-diquark wavefunctions lead to $\frac{\delta\Gamma}{\Gamma} \approx 30\%$ in the case of doubly charmed baryons and $\frac{\delta\Gamma}{\Gamma} \approx 15\%$ for the bcq - baryons.

Thus, the estimated uncertainty in predictions for the lifetimes of doubly heavy baryons is close to 25% in the case of (bcq) - baryons, of order of 45% in the case of doubly charmed baryons and less then 5% in the case of (bbq) - baryons.

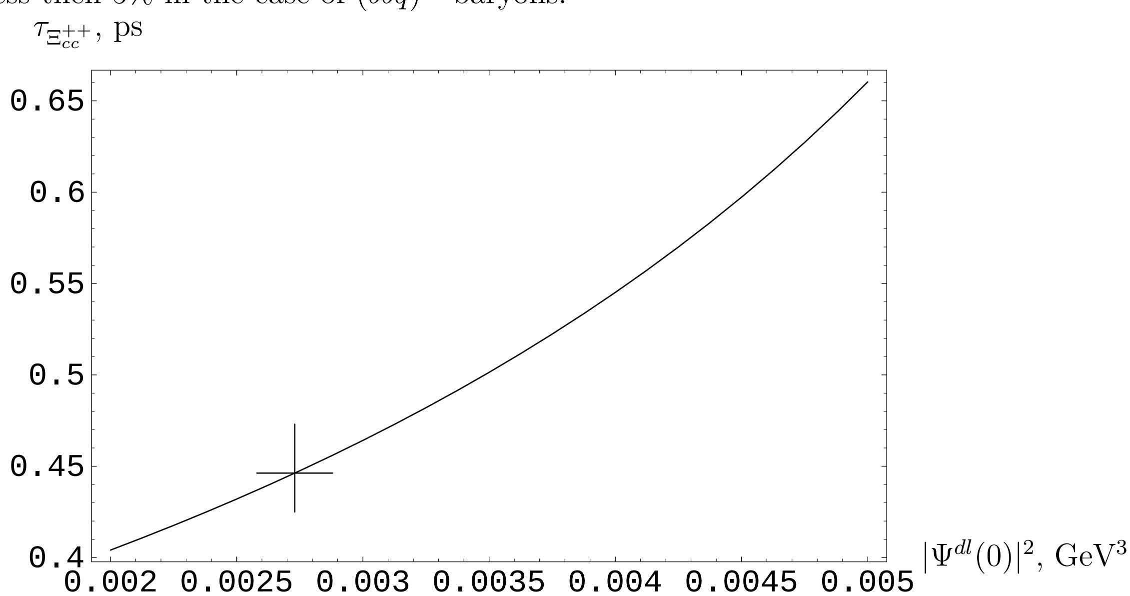

Figure 1: The dependence of Ξ_{cc}^{++}-baryon lifetime on the value of wavefunction of light quark-diquark system at the origin $|\Psi^{dl}(0)|$.

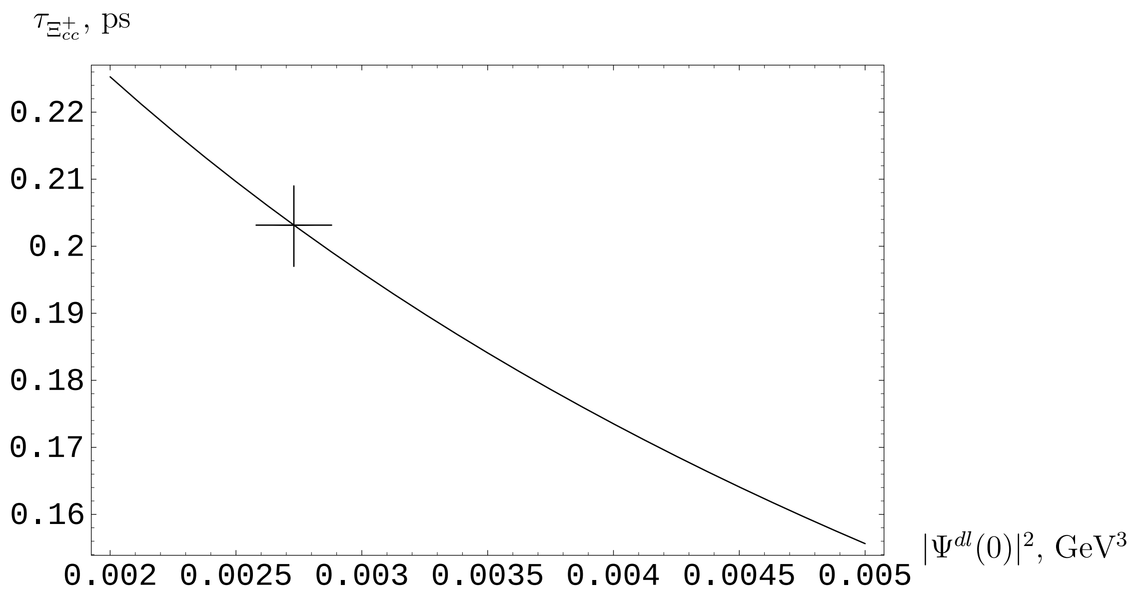

Figure 2: The dependence of Ξ_{cc}^{+}-baryon lifetime on the value of wavefunction of light quark-diquark system at the origin $|\Psi^{dl}(0)|$.

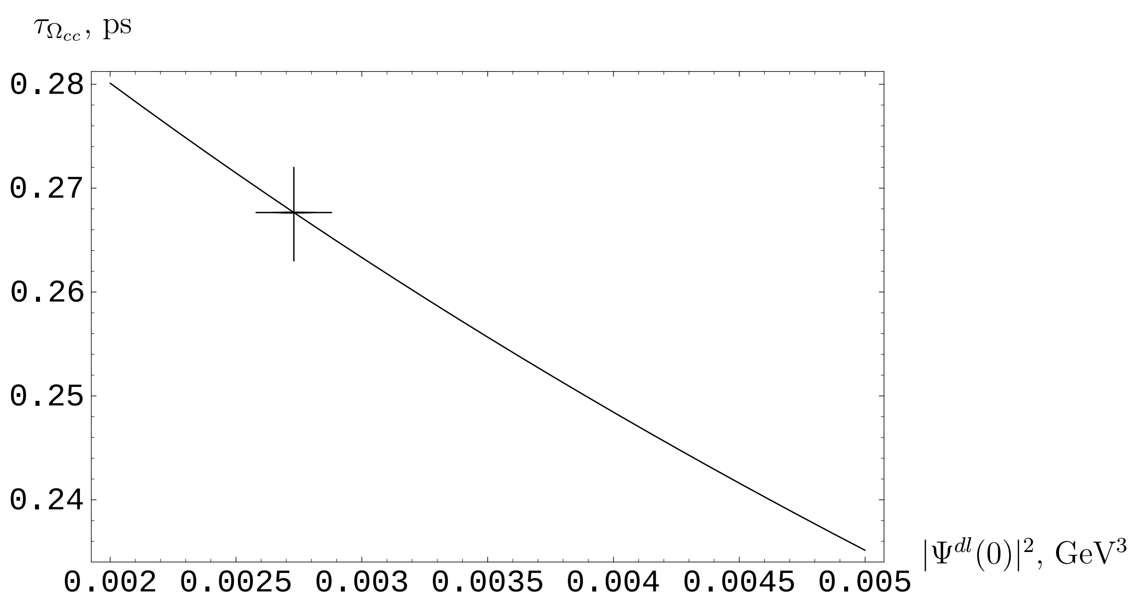

Figure 3: The dependence of Ω_{cc}-baryon lifetime on the value of wavefunction of light quark-diquark system at the origin $|\Psi^{dl}(0)|$.

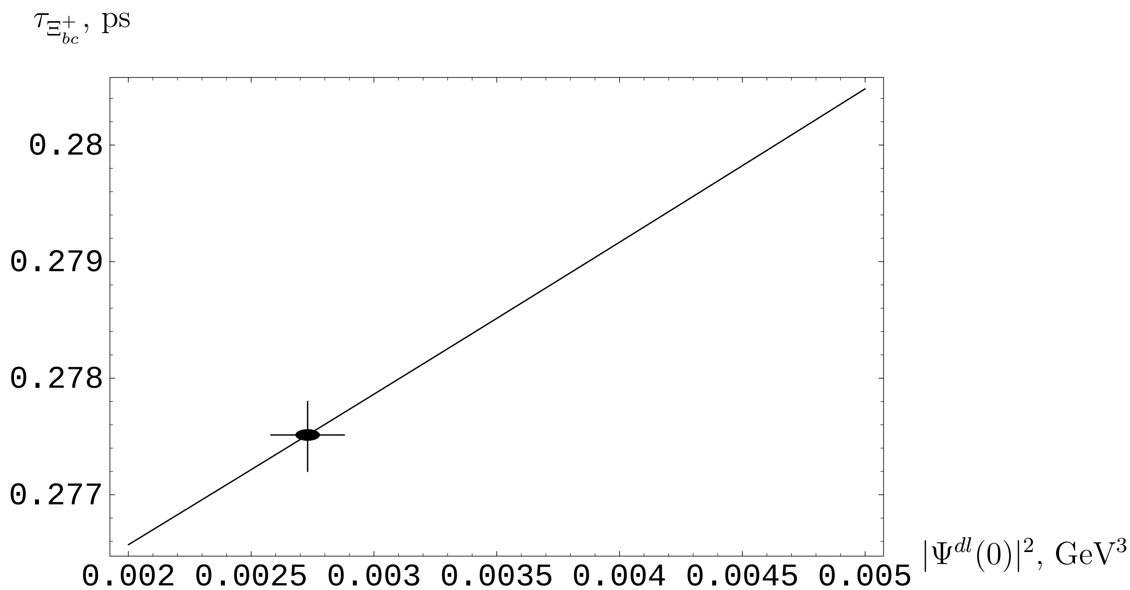

Figure 4: The dependence of Ξ_{bc}^+-baryon lifetime on the value of wavefunction of light quark-diquark system at the origin $|\Psi^{dl}(0)|$.

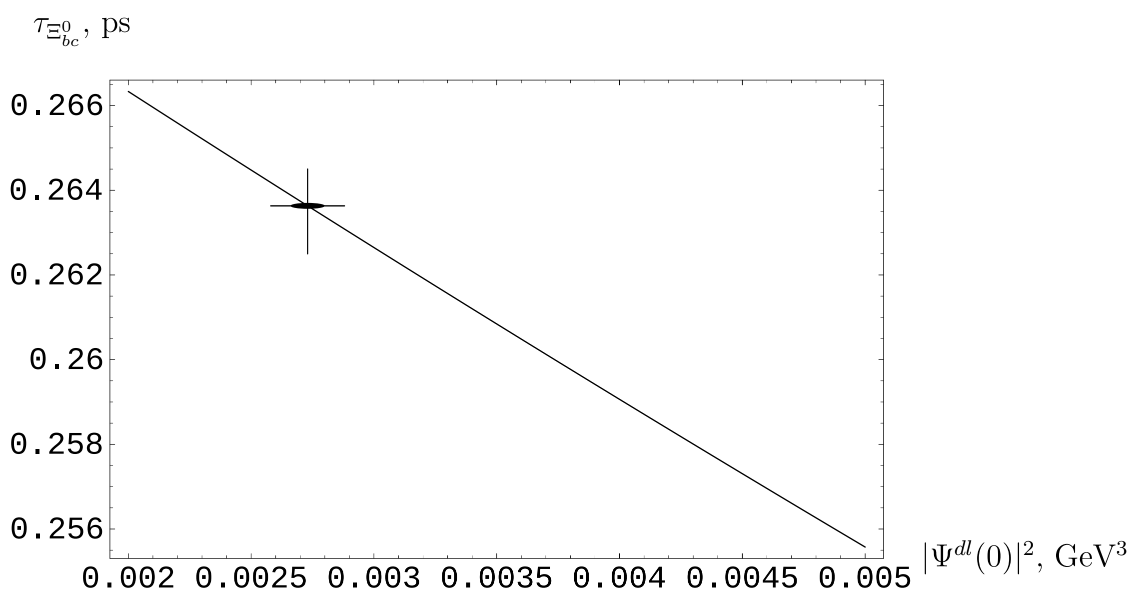

Figure 5: The dependence of Ξ_{bc}^0-baryon lifetime on the value of wavefunction of light quark-diquark system at the origin $|\Psi^{dl}(0)|$.

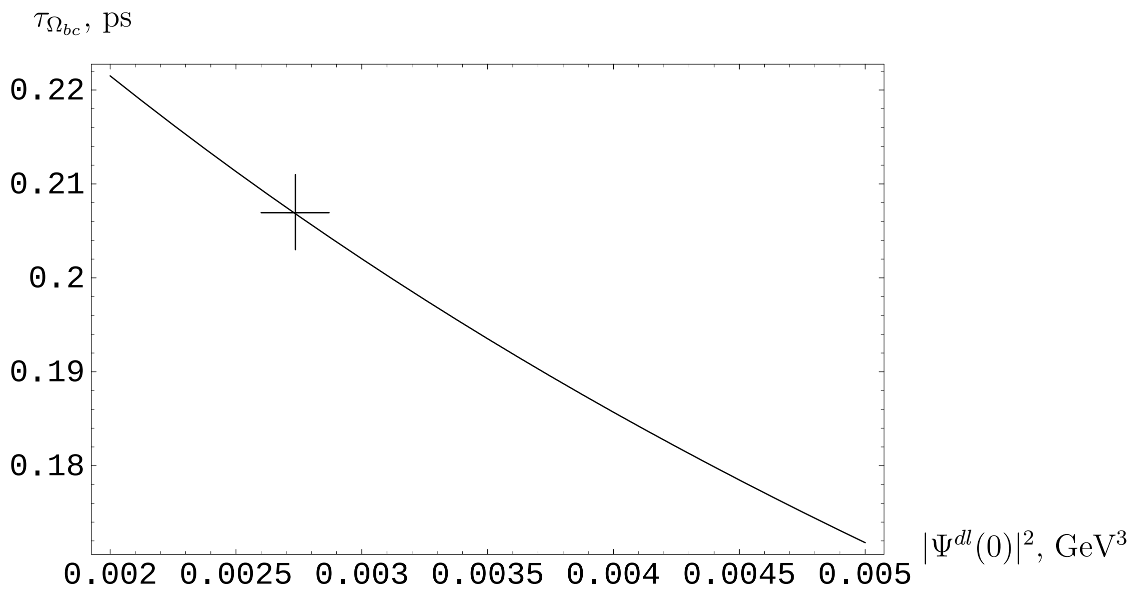

Figure 6: The dependence of Ω_{bc}-baryon lifetime on the value of wavefunction of light quark-diquark system at theorigin $|\Psi^{dl}(0)|$.

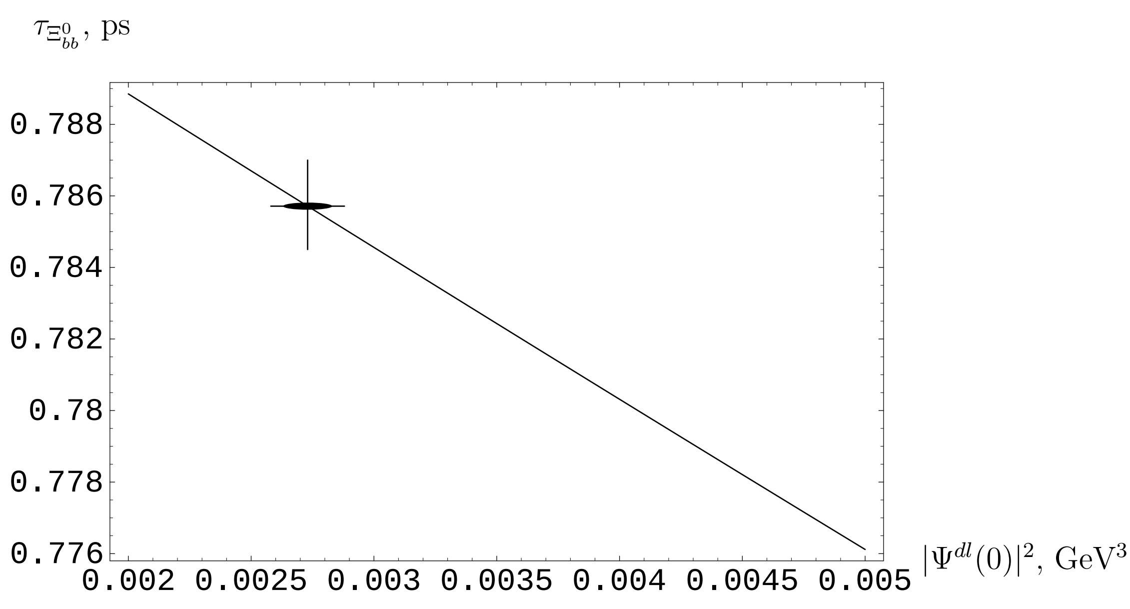

Figure 7: The dependence of Ξ^0_{bb}-baryon lifetime on the value of wavefunction of light quark-diquark system at the origin $|\Psi^{dl}(0)|$.

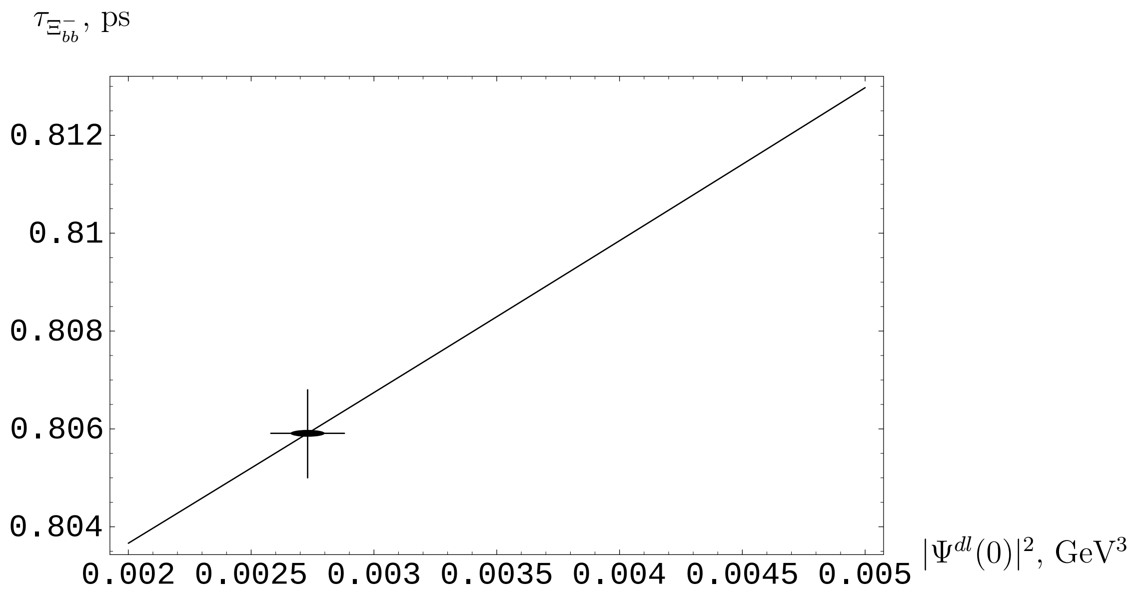

Figure 8: The dependence of Ξ^-_{bb}-baryon lifetime on the value of wavefunction of light quark-diquark system at the origin $|\Psi^{dl}(0)|$.

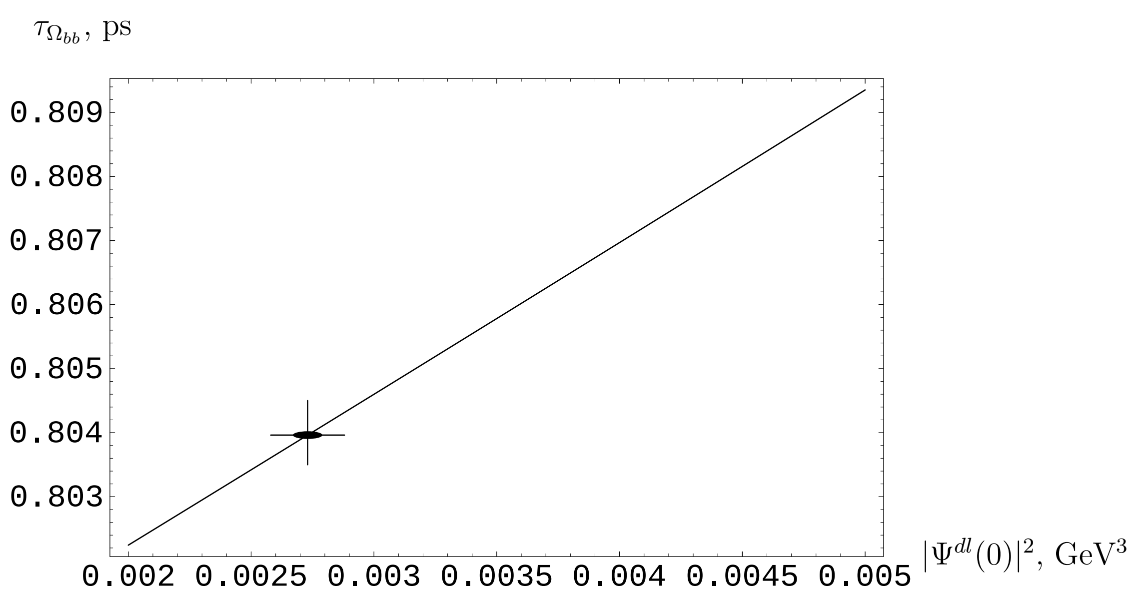

Figure 9: The dependence of Ω_{bb}-baryon lifetime on the value of wavefunction of light quark-diquark system at the origin $|\Psi^{dl}(0)|$.

5 Conclusion

In the present paper we have performed a detail investigation and numerical estimates for the lifetimes of doubly heavy baryons. The used approach is based on OPE expansion of total widths for the corresponding hadrons, and it is combined with the formalism of effective fields theories developed previously. In this way, we have accounted for the both perturbative QCD and mass corrections to the Wilson coefficients of operators. The nonspectator effects, presented by Pauli interference and weak scattering, and their influence on the total lifetimes are considered. The obtained results show the significant role played by them in the description of lifetimes of doubly heavy baryons.

The authors thank V.V.Kiselev for fruitful discussions and remarks concerning the presentation of results.

This work is in part supported by the Russian Foundation for Basic Research, grants 96-02-18216 and 96-15-96575. The work of A.I. Onishchenko was supported by International Center of Fundamental Physics in Moscow and Soros Science Foundation.

References

[1] *A.Ali, D.London*, Nucl. Phys. Proc. Suppl. **54A**, 297 (1997).

[2] *M.Neubert*, Phys. Rep. 245 (1994) 259.

[3] *G.T.Bodwin, E.Braaten, G.P.Lepage*, Phys. Rev. D51 (1995) 1125,
Phys. Rev. D55 (1997) 5853.

[4] *M.Beneke, G.Buchalla*, Phys. Rev. D53 (1996) 4991.

[5] *F.Abe et al.*, Phys. Rev. Lett. 81 (1998) 2432, Phys. Rev. D58 (1998) 112004.

[6] *S.S.Gershtein et al.*, preprint IHEP 98-22. 1998. [hep-ph/9803433];
S.S.Gershtein et al., Uspekhi Fiz. Nauk. 165 (1995) 3.

[7] *V.V. Kiselev, A.K. Likhoded, A.I. Onishchenko*, Phys.Rev. **D60**, (1999) 014007, hep-ph/9807354.

[8] *V.V. Kiselev, A.K. Likhoded, A.I. Onishchenko*, hep-ph/9901224.

[9] *A.I. Onishchenko*, hep-ph/9912424

[10] *B. Guberina, B. Melic, H. Stefancic*, Eur.Phys.J. **C9**, (1999) 213, hep-ph/9901323, hep-ph/9911241.

[11] *S.S. Gershtein, V.V. Kiselev, A.K. Likhoded, A.I. Onishchenko*, Mod.Phys.Lett. **A14**, (1999) 135, hep-ph/9807375

[12] *S.S. Gershtein, V.V. Kiselev, A.K. Likhoded, A.I. Onishchenko*, Heavy Ion Phys. **9**, (1999) 133, hep-ph/9811212

[13] *D.Ebert, R.N.Faustov, V.O.Galkin, A.P.Martynenko, V.A.Saleev*, Z. Phys. C76 (1997) 111;
J.G.Körner, M.Krämer, D.Pirjol, Prog. Part. Nucl. Phys. 33 (1994) 787;
R.Roncaglia, D.B.Lichtenberg, E.Predazzi, Phys. Rev. D52 (1995) 1722;
E.Bagan, M.Chabab, S.Narison, Phys. Lett. B306 (1993) 350;
M.J.Savage and M.B.Wise, Phys. Lett. B248 (1990) 117;
M.J.Savage and R.P.Springer, Int. J. Mod. Phys. A6 (1991) 1701;
S.Fleck and J.M.Richard, Part. World 1 (1989) 760, Prog. Theor. Phys. 82 (1989) 760;
D.B.Lichtenberg, R. Roncaglia, E. Predazzi, Phys. Rev. D53 (1996) 6678;
M.L.Stong, hep-ph/9505217;
J.M.Richard, Phys. Rept. 212 (1992) 1.

[14] *E. Bagan, M. Chabab, S. Narison*, Phys.Lett.**B306**, (1993) 350;
E. Bagan at al., Z.Phys. **C64**, (1994), 57;
V.V. Kiselev, A.I. Onishchenko, hep-ph/9909337.

[15] *V.V.Kiselev, A.K.Likhoded, M.V.Shevlyagin*, Phys. Lett. B332 (1994) 411;
A.Falk et al., Phys. Rev. D49 (1994) 555;
A.V. Berezhnoi, V.V. Kiselev, A.K. Likhoded, Phys. Atom. Nucl. 59 (1996) 870 [Yad. Fiz. 59 (1996) 909];
M.A. Doncheski, J. Steegborn, M.L. Stong, Phys. Rev. D53 (1996) 1247;
S.P.Baranov, Phys. Rev. D56 (1997) 3046;
A.V.Berezhnoy, V.V.Kiselev, A.K.Likhoded, A.I.Onishchenko, Phys. Rev. D57 (1997) 4385;
V.V.Kiselev, A.E.Kovalsky, hep-ph/9908321.

[16] *M.B.Voloshin, M.A.Shifman*, Yad. Fiz. 41 (1985) 187;
M.B.Voloshin, M.A.Shifman, Zh. Exp. Teor. Fiz. 64 (1986) 698;
M.B.Voloshin, preprint TIP-MINN-96/4-T, UMN-TH-1425-96. 1996. [hep-th/9604335].

[17] *I.Bigi et al.*, "B Decays", Second edition, ed. S. Stone (World Scientific, Singapore, 1994)

[18] *B. Guberina, R. Rückl and J. Trampetič̃*, Z.Phys. **C33**, (1986) 297.

[19] *G.P.Lepage et al.*, Phys. Rev. D46 (1992) 4052.

[20] *I.Bigi, N.Uraltsev, A.Vainshtein*, Phys. Lett. B293 (1992) 430, Phys. Lett. B297 (1993) 477;
B.Blok, M.Shifman, Nucl. Phys. B399 (1993) 441, 459;
I.Bigi et al., Phys. Lett. B323 (1994) 408.

[21] *A.V.Manohar, M.B.Wise*, Phys. Rev. D49 (1994)1310.

[22] *A.F.Falk et al.*, Phys. Lett. B326 (1994) 145;
L.Koyrakh, Phys. Rev. D49 (1994) 3379.

[23] *G.Altarelli et al.*, Nucl. Phys. B187 (1981) 461.

[24] *A.J.Buras, P.H.Weisz*, Nucl. Phys. B333 (1990) 66.

[25] *G.Buchalla*, Nucl. Phys. B391 (1993) 501.

[26] *Q.Hokim, X.Y.Pham*, Phys. Lett. B122 (1983) 297, Ann. Phys. 155 (1984) 202.

[27] *E.Bagan et al.*, Nucl. Phys. B432 (1994) 3, Phys. Lett. B342 (1995)362;
E.Bagan et al., Phys. Lett. B351 (1995) 546.

[28] *A. De Rújula, H. Georgi, S.L. Glashow*, Phys. Rev. **D12**, (1975) 147

[29] *J.L. Cortes, J. Sanchez Guillen*, Phys.Rev. **D24**, (1981) 2982.

[30] *A.I. Onishchenko*, in preparation